DEVELOPING THE MATHEMATICALLY PROFICIENT STUDENT

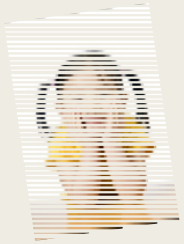
"An understanding can never be 'covered' if it is to be understood."

Wiggins and McTighe 2005

17

If not on grade level by 3rd grade, odds of ever being on grade level are 1 in **17**.

In 4th grade, students need 2 hours of instructional time to make the same gains as made in 30 minutes of instructional time in Kindergarten.



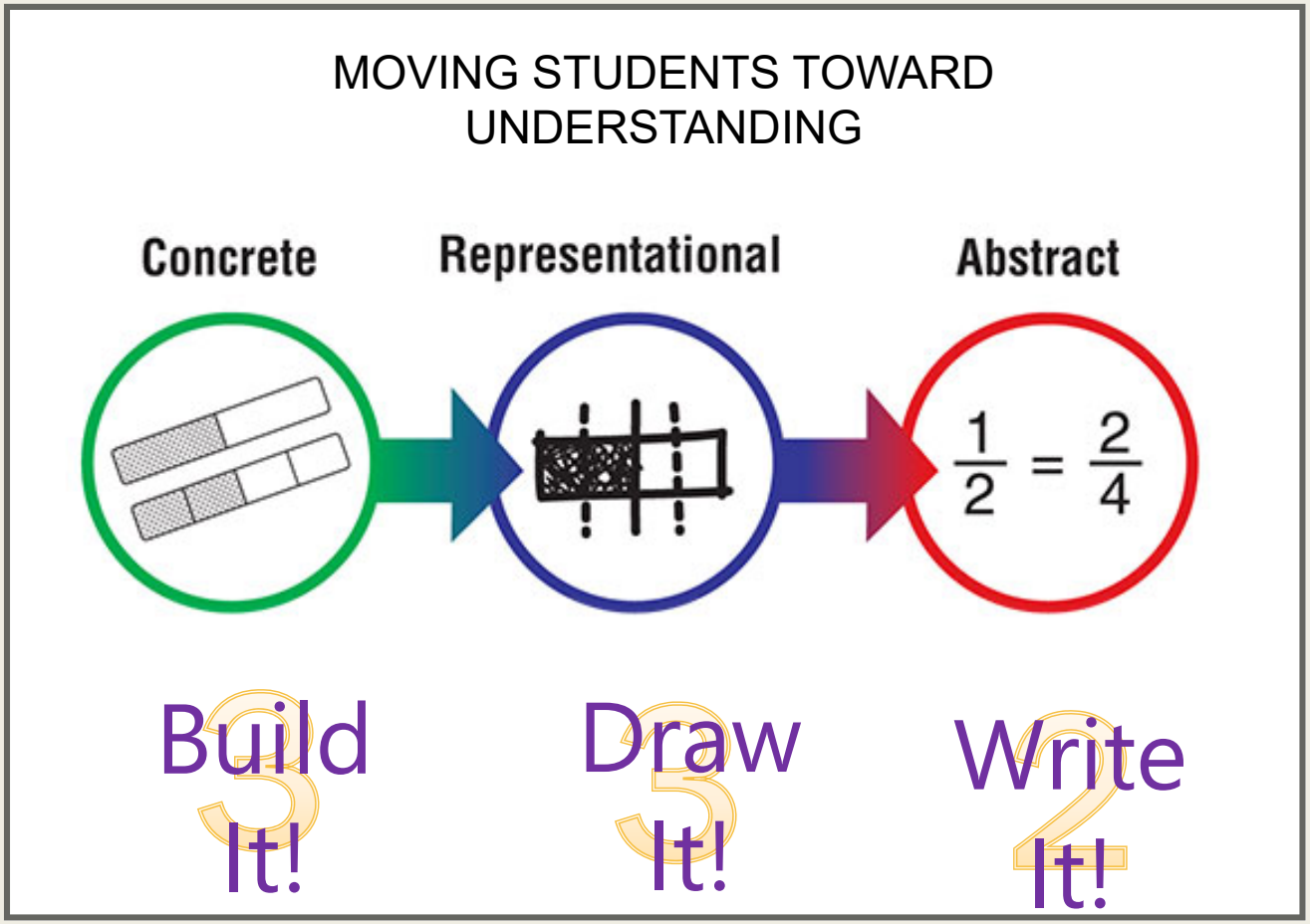
We believe……

- that all students *can* and *should* learn math!

- that if children *like* math and *feel successful* at math -they will learn math!

- having students be involved in "math happenings is critical!

- developing an environment of a "growth mindset" is essential!



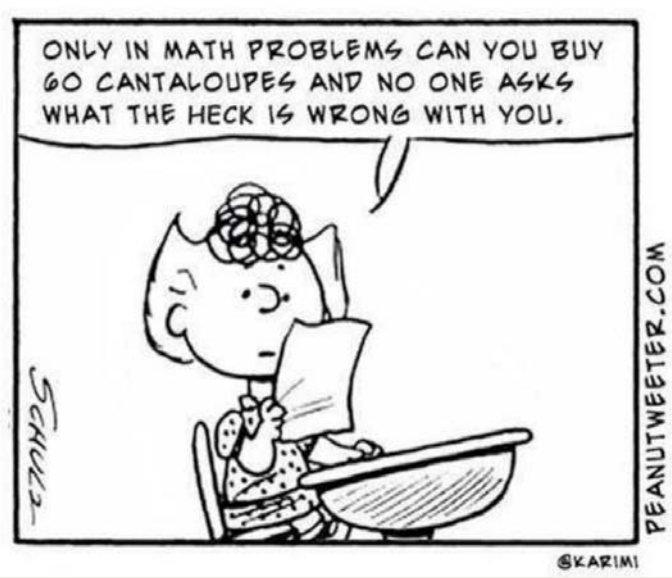
Talk about Math

The math proficient student…

- Understands the concept
- Solves problems using efficient strategies
- Recognizes when and why to use a procedure
- Defends and justifies an answer
- Sees math as challenging and engaging.

KAS

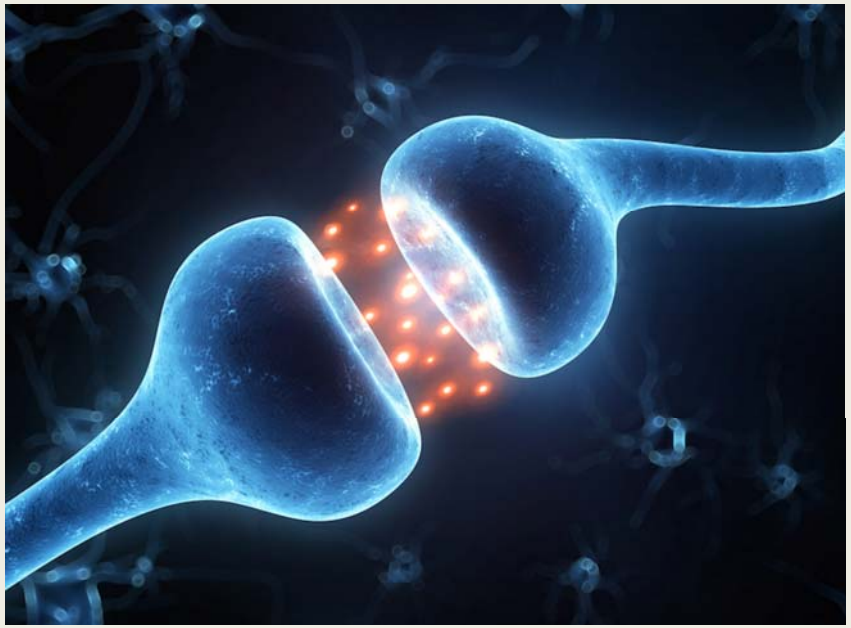
KENTUCKY ACADEMIC STANDARDS



Where are the problems?

The Essential Components of Classroom Instruction

- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition



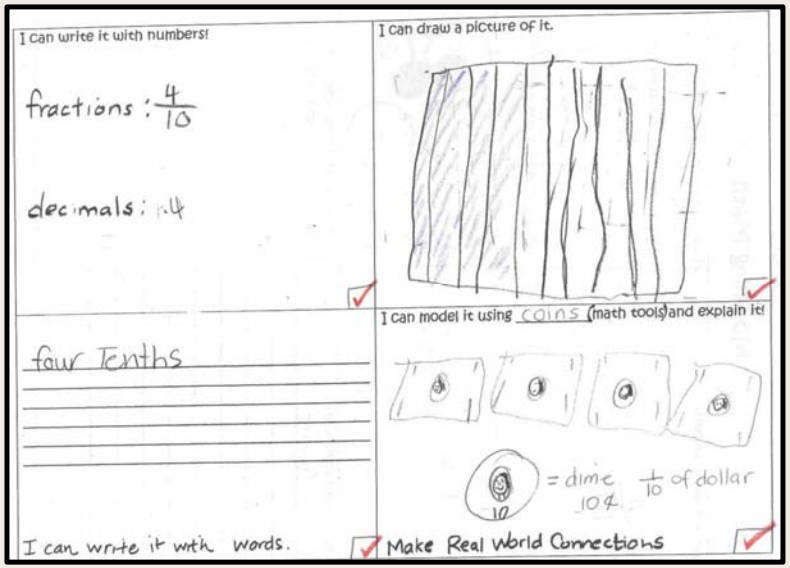
National Resource Council 2001

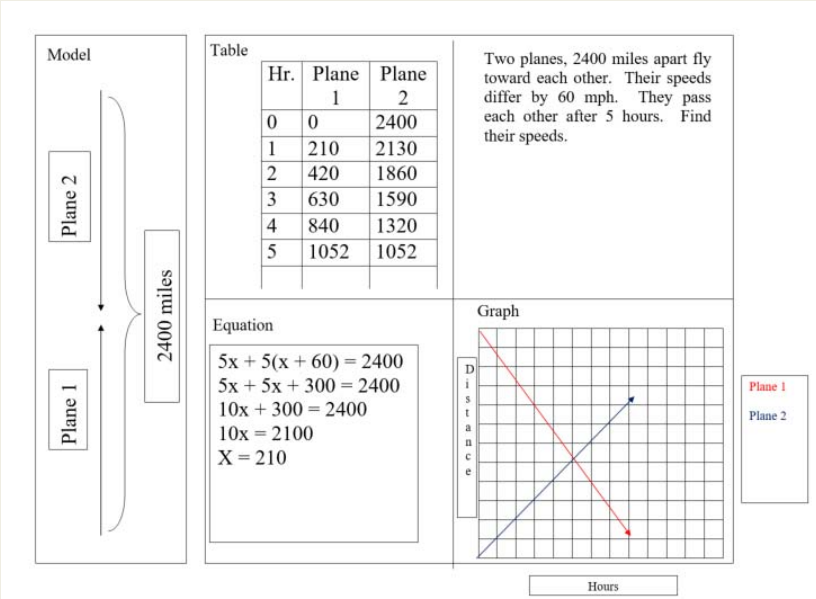
CONCEPTUAL UNDERSTANDING

- Students learn new ideas by connecting to those ideas they already know.
- Supports retention by promoting understanding not memorization
- Encouraging students to ask, "Does my answer make sense?"

National Resource Council 2001

Using Multiple Representations to Demonstrate Understanding.





PROCEDURAL FLUENCY

~*Knowledge of procedures, when and how to use them appropriately and skill in performing them flexibly, accurately and efficiently.*

Thou must learn
to
Borrow and Carry.

National Resource Council 2001

Conceptual Understanding
vs
Procedural Fluency

6 x 7 = ?

5 x 7 = 35

1 x 7 = 7

42

1000 – 196 =

-1 -1

999 – 195 =

GOOD MATH ALWAYS STARTS WITH A QUESTION?

At Kroger's, bananas sells for $.59 per pound. This is ten cents less than at Wal-Mart. How much do 5 pounds of bananas cost at Wal-Mart?

- What do I know?
- What else do I know?
- What do I need to find out?

National Resource Council 2001

Strategic Competence

Subtract $.10 from $.59

At Kroger's, bananas sells for $.59 per pound. This is ten cents less than at Wal-mart. How much do 5 pounds of bananas cost at Wal-mart?

Total

5 x .49 = $2.45

5 x .69 = $ 3.45

ADAPTIVE REASONING

- Students need to be able to justify and explain their ideas in order to…
  - *Make their reasoning clear*
  - *Hone their reasoning skills*
  - *Improve conceptual understanding*

$$3.42 \times 4.90 = 16381$$

National Resource Council 2001

PRODUCTIVE DISPOSTION

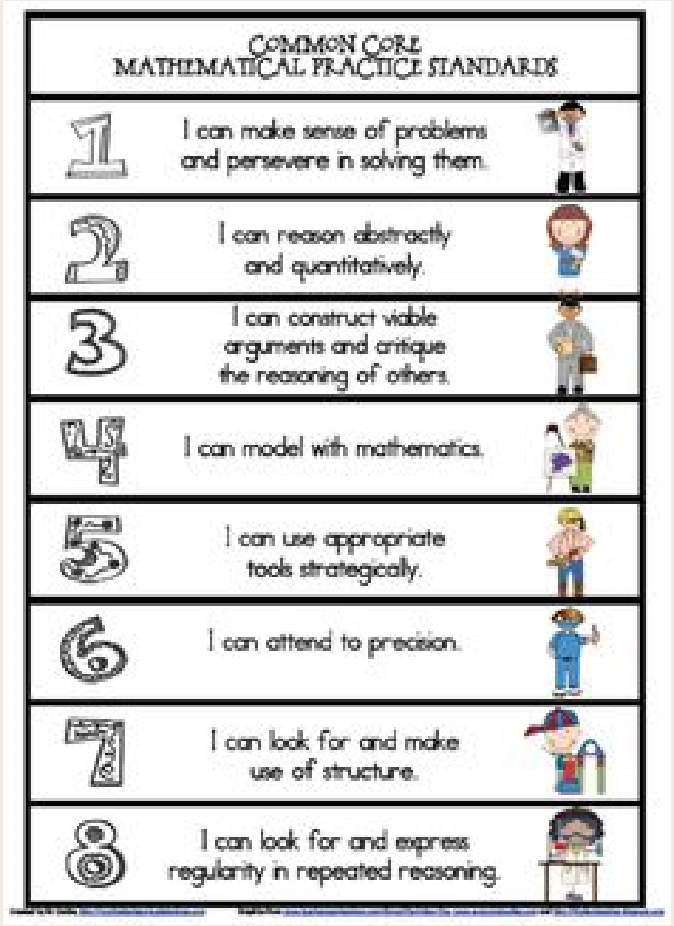
~Capacity to think logically ~ see mathematics as sensible, useful and worthwhile.

National Resource Council 2001

- The Math Gene
- "Math Happenings" Suh, 2007
- The Fixed Mind-set:

The Standards of Mathematical Practice

What our students will do to demonstrate proficiency.



It starts with a question

Common Core State Standards
Standards for Mathematical Practice
Questions for Teachers to Ask

Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics
Teachers ask: • What is this problem asking? • How could you start this problem? • How could you make this problem easier to solve? • How is ___'s way of solving the problem like/different from yours? • Does your plan make sense? Why or why not? • What tools/manipulatives might help you? • What are you having trouble with? • How can you check this?	*Teachers ask:* • What does the number ____ represent in the problem? • How can you represent the problem with symbols and numbers? • Create a representation of the problem.	*Teachers ask:* • How is your answer different than _____'s? • How can you prove that your answer is correct? • What math language will help you prove your answer? • What examples could prove or disprove your argument? • What do you think about _____'s argument • What is wrong with ____'s thinking? • What questions do you have for ____? **it is important that the teacher poses tasks that involve arguments or critiques*	*Teachers ask:* • Write a number sentence to describe this situation • What do you already know about solving this problem? • What connections do you see? • Why do the results make sense? • Is this working or do you need to change your model? **It is important that the teacher poses tasks that involve real world situations*
Use appropriate tools strategically	**Attend to precision**	**Look for and make use of structure**	**Look for and express regularity in repeated reasoning**
Teachers ask: • How could you use manipulatives or a drawing to show your thinking? • Which tool/manipulative would be best for this problem? • What other resources could help you solve this problem?	*Teachers ask:* • What does the word ____ mean? • Explain what you did to solve the problem. • Compare your answer to _____'s answer • What labels could you use? • How do you know your answer is accurate? • Did you use the most efficient way to solve the problem?	*Teachers ask:* • Why does this happen? • How is ____ related to ____? • Why is this important to the problem? • What do you know about ____ that you can apply to this situation? • How can you use what you know to explain why this works? • What patterns do you see? **deductive reasoning (moving from general to specific)*	*Teachers ask:* • What generalizations can you make? • Can you find a shortcut to solve the problem? How would your shortcut make the problem easier? • How could this problem help you solve another problem? **inductive reasoning (moving from specific to general)*

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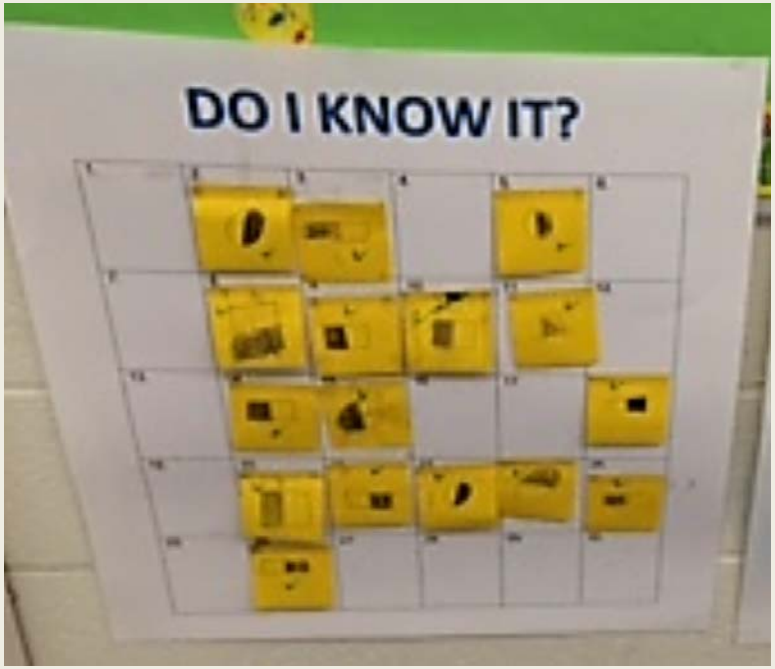
CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS

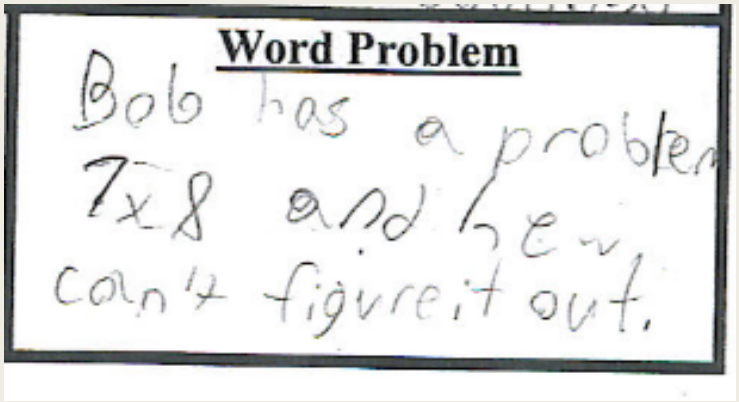


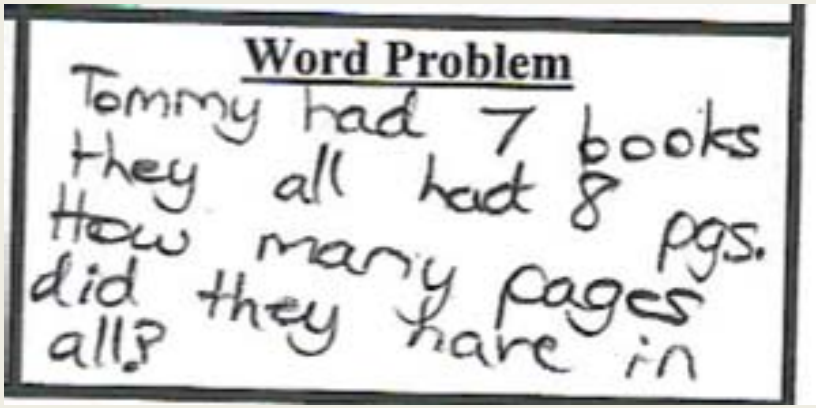
Three Levels of Convincing*

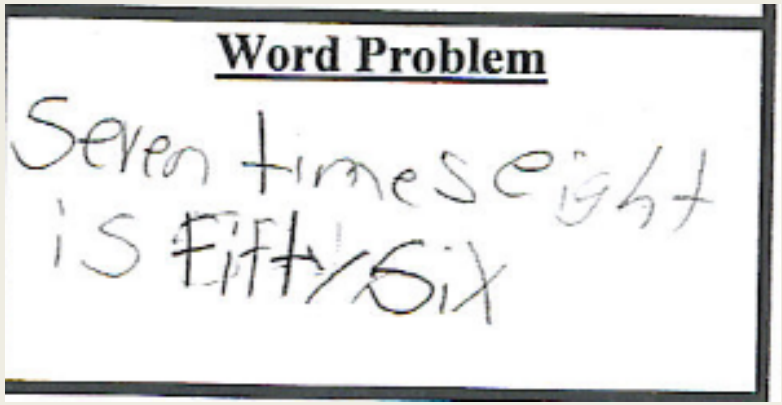
~Convince Yourself
~Convince a Friend
~Convince a Skeptic

*Boaler and Humphreys, 2005

7x8 = 56



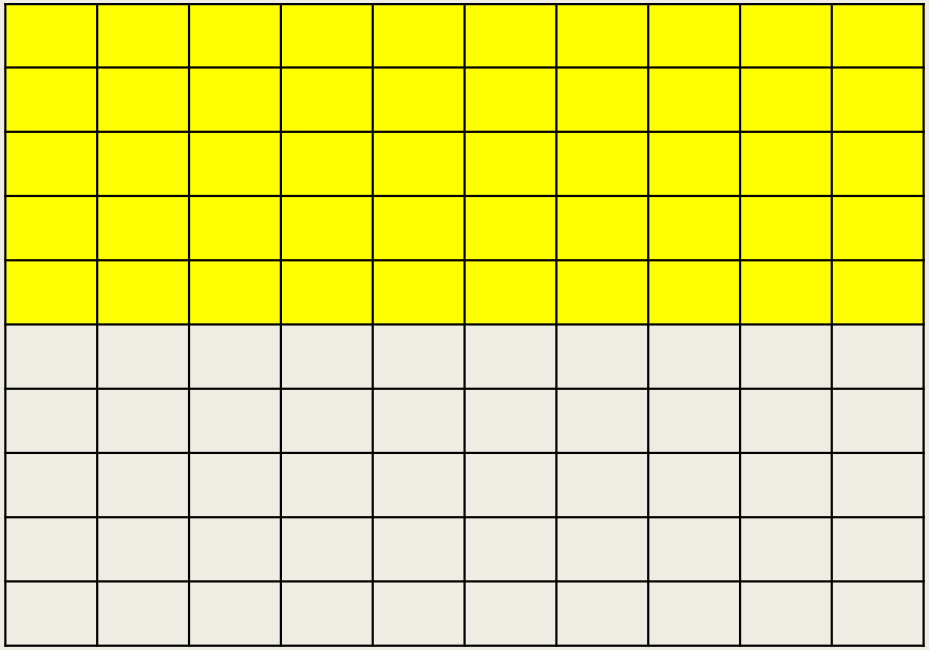




MODEL WITH MATHEMATICS

USE APPROPRIATE TOOLS STRATEGICALLY (not just a calculator)

- Students consider the available tools when solving a problem.



$$\frac{1}{2} = \frac{5}{10} = \frac{50}{100} = .50$$

Fraction to Decimal Relationship

ATTEND TO PRECISION

- Students communicate their ideas through the use of clear definitions with others and with their own reasoning.

What vocabulary will I use to express my answer?

LOOK FOR AND MAKE USE OF STRUCTURE

- Do I see a pattern ?
- Does this work like other problems I have done?

$$13 + 29 + 77 + 11 = (13 + 29) + (29 + 11)$$

$$3^2 + 2(6-3) + \frac{6+8}{7}$$

$$9 + 2(3) + \frac{14}{7}$$

$$9 + 6 + 2$$

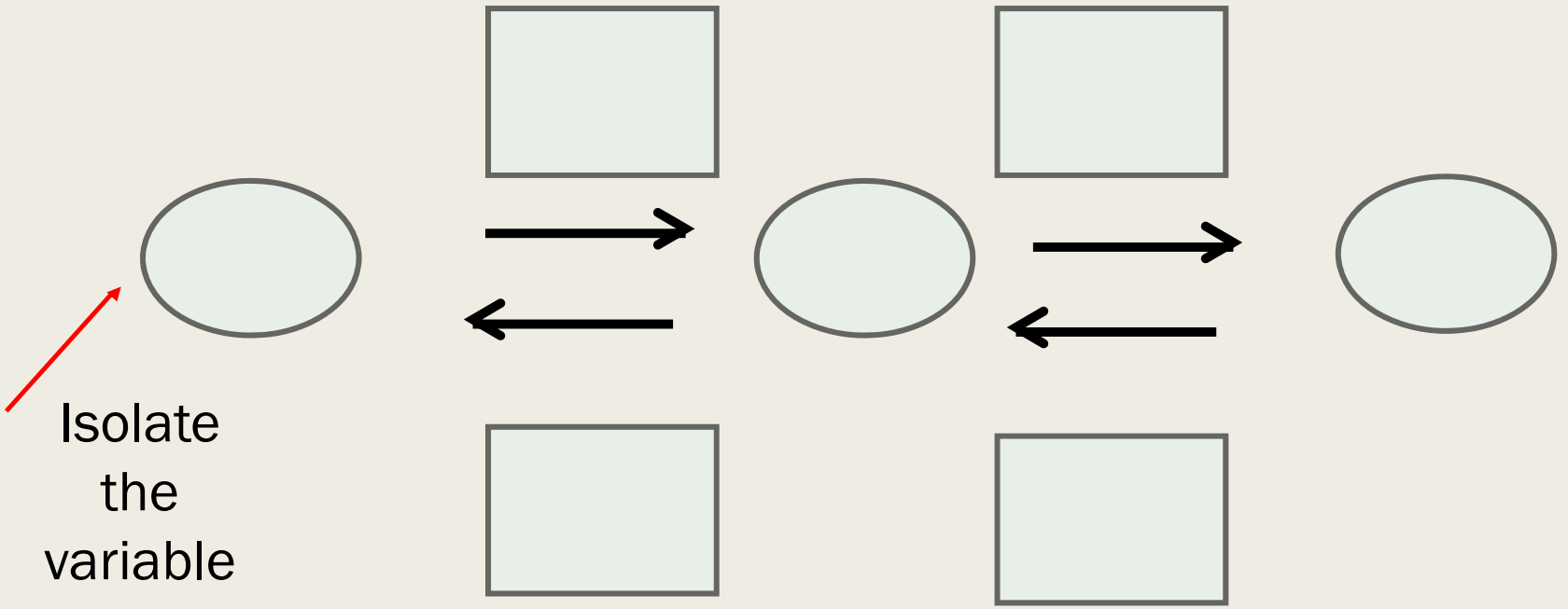
$$15 + 2$$

$$17$$

LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING

$4n + 10 = 50$

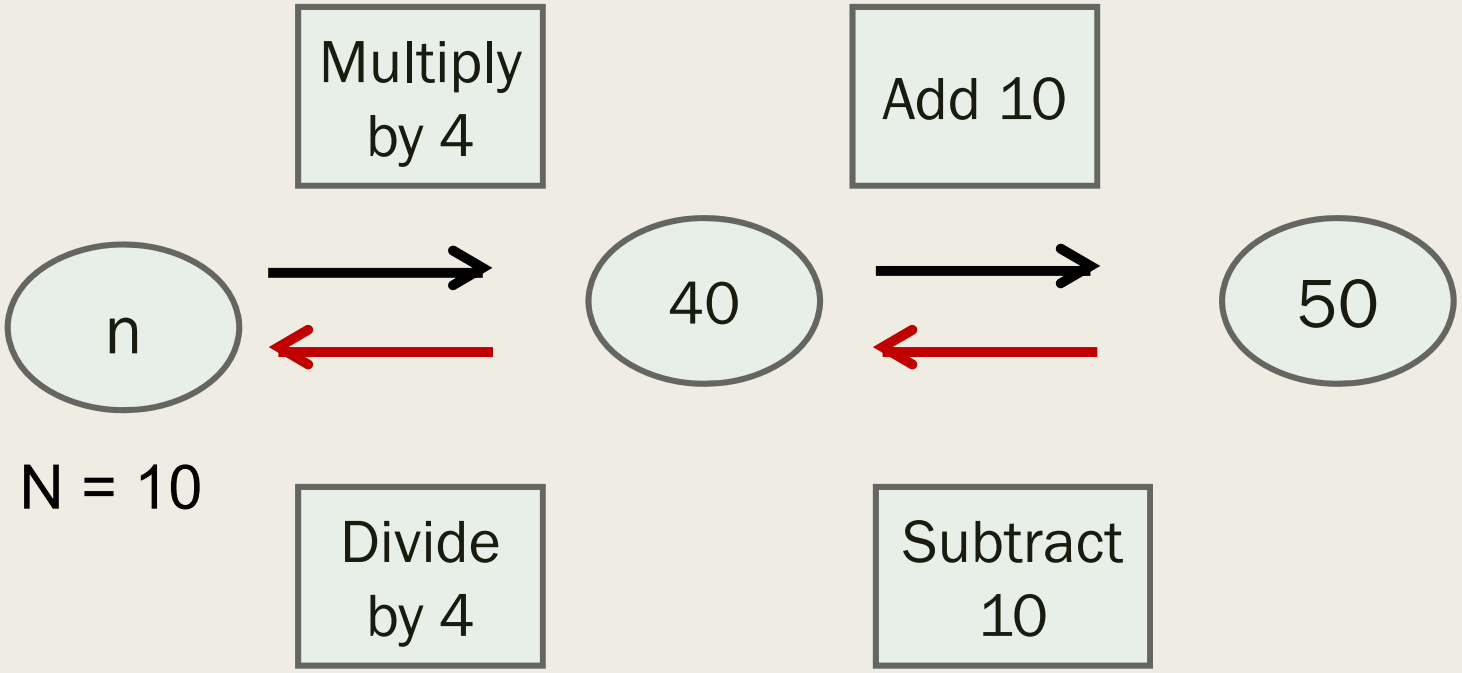
Breakdown the Problem



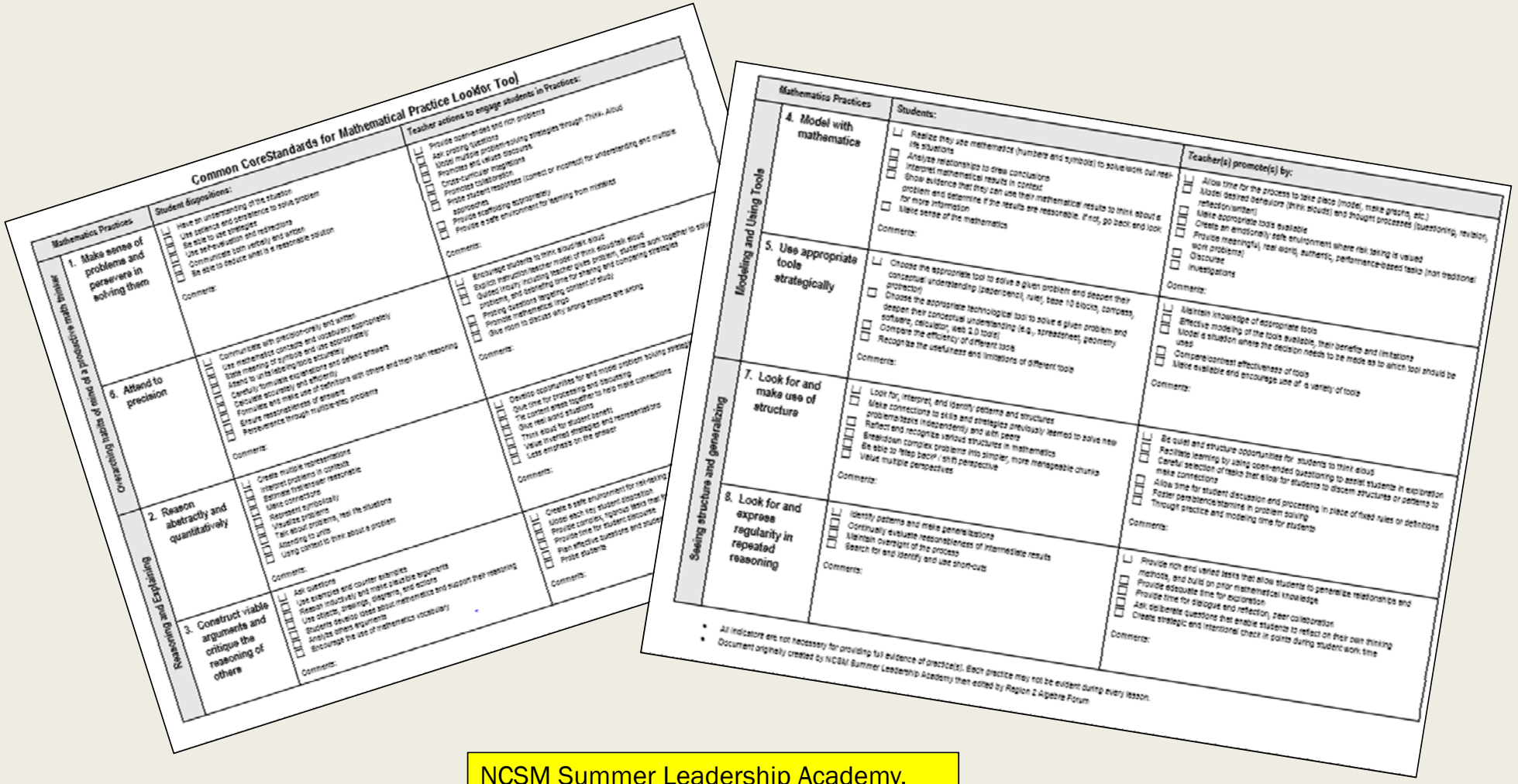
Isolate the variable

Work Backwards to Solve Using Inverse Operations

4n + 10 = 50



Handout – *Look-For Tool*



Common Core Standards for Mathematical Practice Look-For Tool

Mathematics Practices	Student dispositions:	Teacher actions to engage students in Practices:
Overarching habits of mind of a productive math thinker: 1. Make sense of problems and persevere in solving them	Have an understanding of the situation Use patience and persistence to solve problem Use strategies Use self-evaluation and reflections Communicate both verbally and written Be able to realize when is a reasonable solution Comments:	Provide open-ended and rich problems Ask probing questions Model multiple problem-solving strategies through Think- Aloud Promotes and values discourse Cross-curricular integrations Promotes collaboration Probe student responses (correct or incorrect) for understanding and multiple approaches Provide scaffolding appropriately Provide a safe environment for learning from mistakes Comments:
6. Attend to precision	Communicate with precision-orally and written Use mathematics concepts and vocabulary appropriately State meaning of symbols and use appropriately Attend to units in stating solutions and criteria answers Carefully formulate explanations and criteria answers Calculate accurately and efficiently Formulate and make use of definitions with others and their own reasoning Ensure reasonableness of answers Persevere through multiple-step problems Comments:	Encourage students to think aloud/self aloud Explicit instruction/teacher model of think aloud/self aloud Guided inquiry including teacher gives problem, students work together to solve problems, and appropriate time for sharing and considering strategies Probing questions targeting content of study Pose math vocabulary terms Give room to discuss why wrong answers are wrong Comments:
Reasoning and Explaining: 2. Reason abstractly and quantitatively	Create multiple representations Interpret problems in contexts Estimate reasonable/ reasonable Make connections Represent symbolically Visualize problems Talk about problems, real life situations Attending to units Using context to think about a problem Comments:	Develop opportunities for and model problem solving strategies Give time for processing and discussing Tie content areas together to help make connections Give real world situations Think aloud for student benefit Value invented strategies and representations Less emphasis on the answer Comments:
3. Construct viable arguments and critique the reasoning of others	Ask questions Use examples and counter examples Reason inductively and make plausible arguments Use objects, drawings, diagrams, and actions Students develop ideas about mathematics and support their reasoning Analyze others arguments Encourage the use of mathematics vocabulary Comments:	Create a safe environment for risk-taking Model each key student disposition Provide complex, rigorous tasks that foster perseverance Provide time for student discourse Plan effective questions and student discourse Probe students Comments:

Mathematics Practices	Students:	Teacher(s) promote(s) by:
Modeling and Using Tools: 4. Model with mathematics	Realize they use mathematics (numbers and symbols) to solve/work out real-life situations Analyze relationships to draw conclusions Interpret mathematical results in context Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable. If not, go back and look for more information Make sense of the mathematics Comments:	Allow time for the process to take place (model, make graphs, etc.) Model desired behaviors (think aloud) and thought processes (questioning, revision, reflection/written) Make appropriate tools available Create an emotionally safe environment where risk taking is valued Provide meaningful, real world, authentic, performance-based tasks (non traditional work problems) Discourse Investigations Comments:
5. Use appropriate tools strategically	Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (pedestrian), ruler, base 10 blocks, compass, protractor Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) Compare the efficiency of different tools Recognize the usefulness and limitations of different tools Comments:	Maintain knowledge of appropriate tools Effective modeling of the tools available, their benefits and limitations Model a situation where the decision needs to be made as to which tool should be used Compare/contrast effectiveness of tools Make available and encourage use of a variety of tools Comments:
Seeing structure and generalizing: 7. Look for and make use of structure	Look for, interpret, and identify patterns and structures Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers Reflect and recognize various structures in mathematics Breakdown complex problems into simpler, more manageable chunks Be able to "step back" / shift perspective Value multiple perspectives Comments:	Be quiet and structure opportunities for students to think aloud Facilitate learning by using open-ended questioning to assist students in exploration Careful selection of tasks that allow for students to discern structures or patterns to make connections Allow time for student discussion and processing in place of fixed rules or definitions Foster perseverance/stamina in problem solving Through practice and modeling time for students Comments:
8. Look for and express regularity in repeated reasoning	Identify patterns and make generalizations Continually evaluate reasonableness of intermediate results Maintain oversight of the process Search for and identify and use short-cuts Comments:	Provide rich and varied tasks that allow students to generalize relationships and methods, and build on prior mathematical knowledge Provide adequate time for exploration Provide time for dialogue and reflection, peer collaboration Ask deliberate questions that enable students to reflect on their own thinking Create strategic and intentional check in points during student work time Comments:

- All indicators are not necessarily for providing full evidence of practice(s). Each practice may not be evident during every lesson.
- Document originally created by NCSM Summer Leadership Academy then edited by Region 2 Algebra Forum

NCSM Summer Leadership Academy,
Region 2, Algebra Forum

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