DEVELOPING THE MATHEMATICALLY PROFICIENT STUDENT

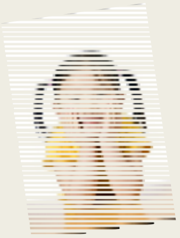
"An understanding can never be 'covered' if it is to be understood."

Wiggins and McTighe 2005

17

If not on grade level by 3rd grade, odds of ever being on grade level are 1 in **17**.

In 4th grade, students need **2** hours of instructional time to make the same gains as made in 30 minutes of instructional time in Kindergarten.



We believe……

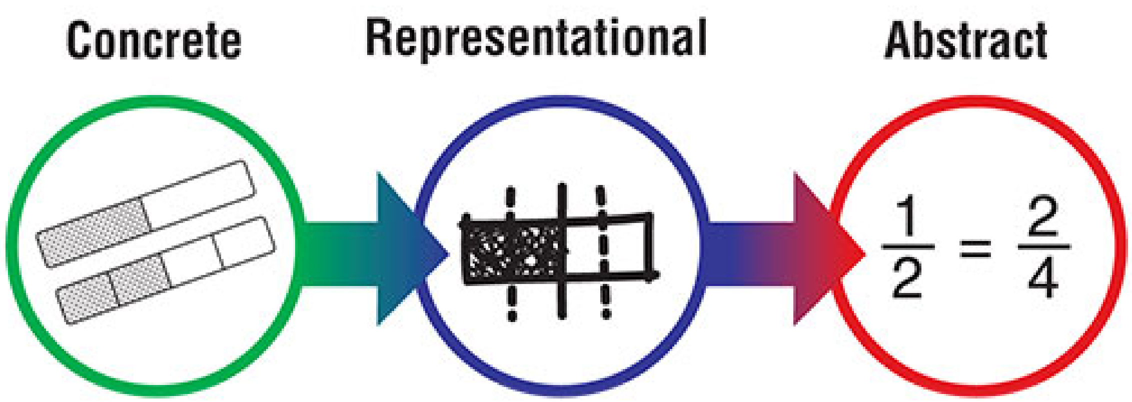
- that all students *can* and *should* learn math!

- that if children *like* math and *feel successful* at math -they will learn math!

- having students be involved in “math happenings is critical!

- developing an environment of a “growth mindset” is essential!

MOVING STUDENTS TOWARD UNDERSTANDING



Build It! 3

Draw It! 3

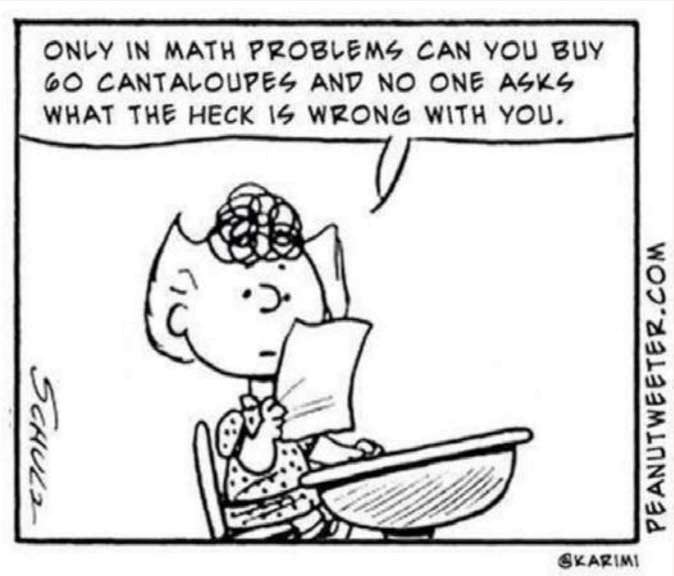
Write It! 2

Talk about Math

The math proficient student…

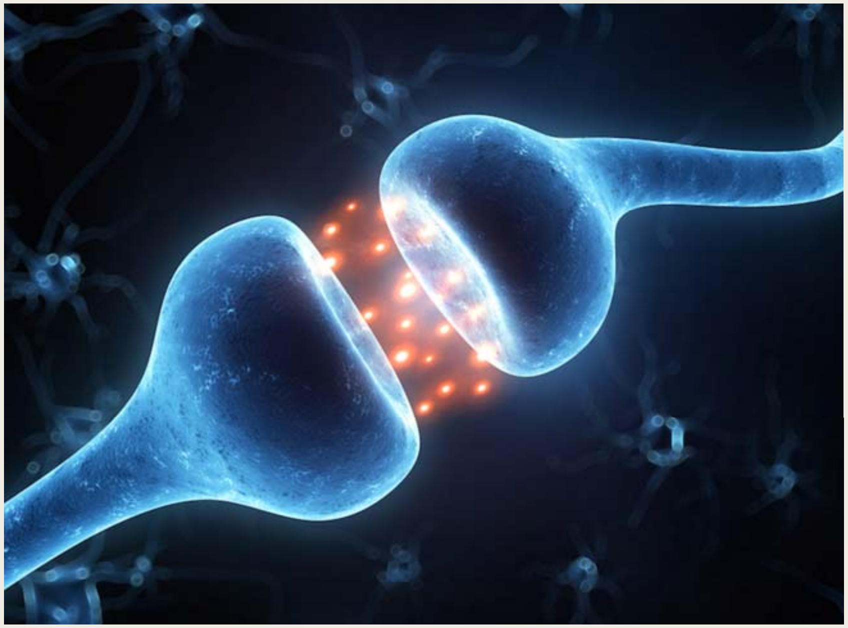
- Understands the concept
- Solves problems using efficient strategies
- Recognizes when and why to use a procedure
- Defends and justifies an answer
- Sees math as challenging and engaging.

KAS
KENTUCKY ACADEMIC STANDARDS



Where are the problems?

The Essential Components of Classroom Instruction



- Conceptual Understanding
- Procedural Fluency
- Strategic Competence
- Adaptive Reasoning
- Productive Disposition

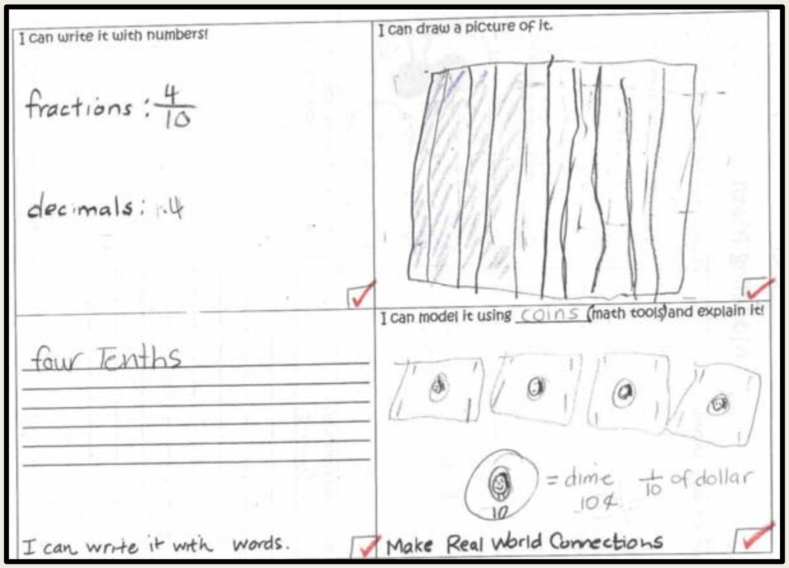
National Resource Council 2001

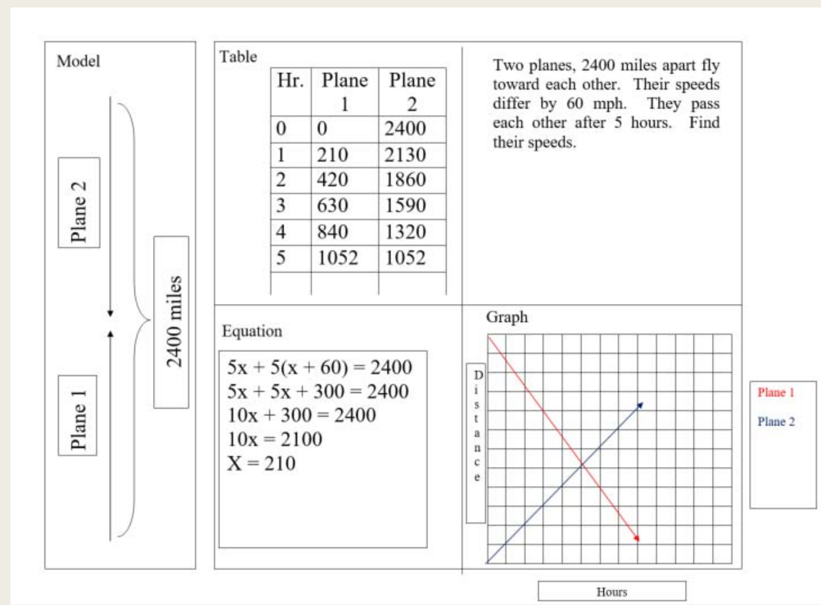
CONCEPTUAL UNDERSTANDING

- Students learn new ideas by connecting to those ideas they already know.
- Supports retention by promoting understanding not memorization
- Encouraging students to ask, "Does my answer make sense?"

National Resource Council 2001

Using Multiple Representations to Demonstrate Understanding.





Hr.	Plane 1	Plane 2
0	0	2400
1	210	2130
2	420	1860
3	630	1590
4	840	1320
5	1052	1052

PROCEDURAL FLUENCY

~*Knowledge of procedures, when and how to use them appropriately and skill in performing them flexibly, accurately and efficiently.*

Thou must learn
to
Borrow and Carry.

National Resource Council 2001

Conceptual Understanding
vs
Procedural Fluency

6 x 7 = ?

5 x 7 = 35
1 x 7 = <u> 7 </u>
42

1000 – 196 =
-1 -1
999 – 195 =

GOOD MATH ALWAYS STARTS WITH A QUESTION?

At Kroger's, bananas sells for $.59 per pound. This is ten cents less than at Wal-Mart. How much do 5 pounds of bananas cost at Wal-Mart?

- What do I know?
- What else do I know?
- What do I need to find out?

National Resource Council 2001

Strategic Competence

Subtract $.10 from $.59

At Kroger's, bananas sells for $.59 per pound. This is ten cents less than at Wal-mart. How much do 5 pounds of bananas cost at Wal-mart?

Total

5 x .49 = $2.45

5 x .69 = $ 3.45

ADAPTIVE REASONING

- Students need to be able to justify and explain their ideas in order to…
  - *Make their reasoning clear*
  - *Hone their reasoning skills*
  - *Improve conceptual understanding*

$$3.42 \times 4.90 = 16381$$

National Resource Council 2001

PRODUCTIVE DISPOSTION

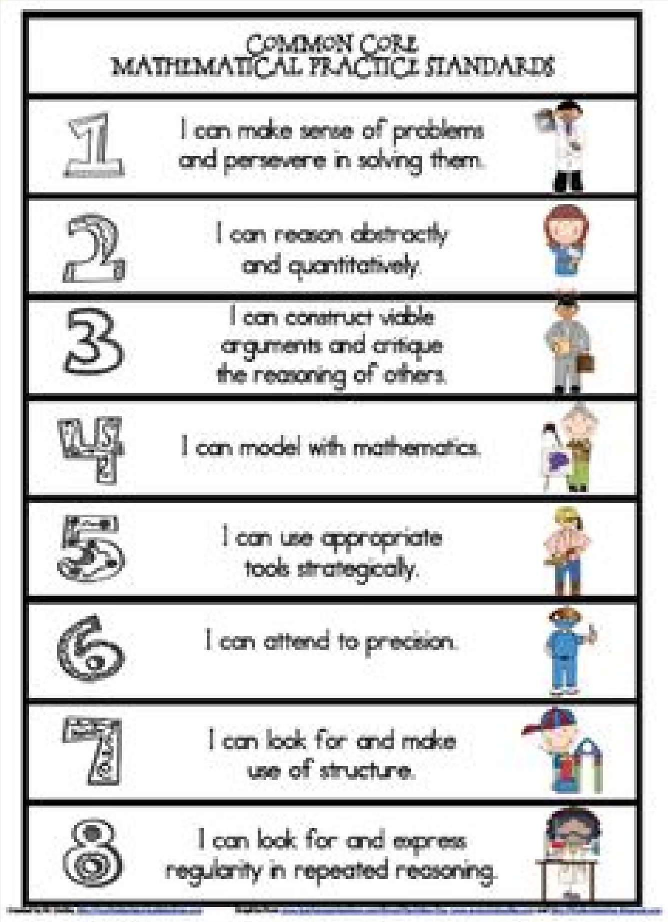
~Capacity to think logically ~ see mathematics as sensible, useful and worthwhile.

National Resource Council 2001

- The Math Gene
- "Math Happenings" Suh, 2007
- The Fixed Mind-set:

The Standards of Mathematical Practice

What our students will do to demonstrate proficiency.



It starts with a question

Common Core State Standards
Standards for Mathematical Practice
Questions for Teachers to Ask

Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics
Teachers ask: • What is this problem asking? • How could you start this problem? • How could you make this problem easier to solve? • How is ___'s way of solving the problem like/different from yours? • Does your plan make sense? Why or why not? • What tools/manipulatives might help you? • What are you having trouble with? • How can you check this?	*Teachers ask:* • What does the number ____ represent in the problem? • How can you represent the problem with symbols and numbers? • Create a representation of the problem.	*Teachers ask:* • How is your answer different than _____'s? • How can you prove that your answer is correct? • What math language will help you prove your answer? • What examples could prove or disprove your argument? • What do you think about _____'s argument • What is wrong with ____'s thinking? • What questions do you have for ____? **it is important that the teacher poses tasks that involve arguments or critiques*	*Teachers ask:* • Write a number sentence to describe this situation • What do you already know about solving this problem? • What connections do you see? • Why do the results make sense? • Is this working or do you need to change your model? **It is important that the teacher poses tasks that involve real world situations*
Use appropriate tools strategically	**Attend to precision**	**Look for and make use of structure**	**Look for and express regularity in repeated reasoning**
Teachers ask: • How could you use manipulatives or a drawing to show your thinking? • Which tool/manipulative would be best for this problem? • What other resources could help you solve this problem?	*Teachers ask:* • What does the word ____ mean? • Explain what you did to solve the problem. • Compare your answer to _____'s answer • What labels could you use? • How do you know your answer is accurate? • Did you use the most efficient way to solve the problem?	*Teachers ask:* • Why does this happen? • How is ____ related to ____? • Why is this important to the problem? • What do you know about ____ that you can apply to this situation? • How can you use what you know to explain why this works? • What patterns do you see? **deductive reasoning (moving from general to specific)*	*Teachers ask:* • What generalizations can you make? • Can you find a shortcut to solve the problem? How would your shortcut make the problem easier? • How could this problem help you solve another problem? **inductive reasoning (moving from specific to general)*

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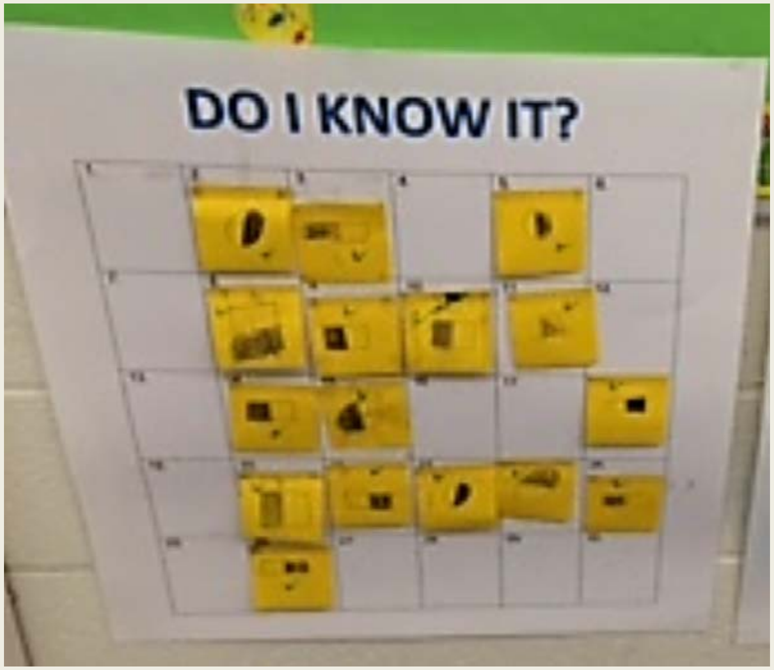
CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS



Three Levels of Convincing*

~Convince Yourself
~Convince a Friend
~Convince a Skeptic

*Boaler and Humphreys, 2005

7x8 = 56

Word Problem

Tommy had 7 books they all had 8 pgs. How many pages did they have in all?

Word Problem

Bob has a problem 7x8 and he can't figure it out.

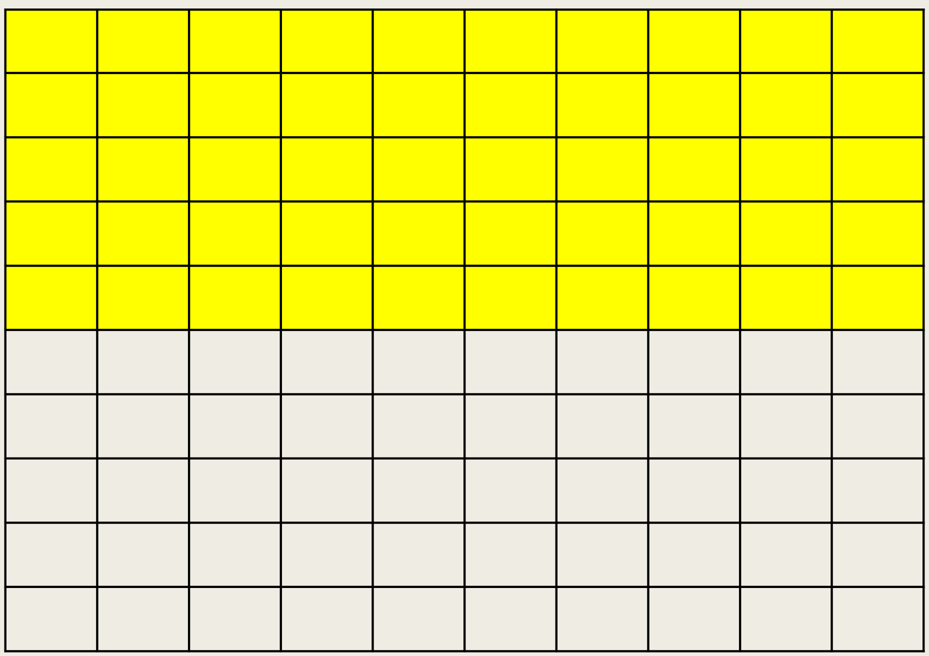
Word Problem

Seven times eight is Fifty six

MODEL WITH MATHEMATICS

USE APPROPRIATE TOOLS STRATEGICALLY
(not just a calculator)

- Students consider the available tools when solving a problem.



$$\frac{1}{2} = \frac{5}{10} = \frac{50}{100} = .50$$

Fraction to Decimal Relationship

ATTEND TO PRECISION

- Students communicate their ideas through the use of clear definitions with others and with their own reasoning.

What vocabulary will I use to express my answer?

LOOK FOR AND MAKE USE OF STRUCTURE

- Do I see a pattern ?
- Does this work like other problems I have done?

13 + 29 + 77 + 11 = (13 + 29) + (29 + 11)

$$3^2 + 2(6-3) + \frac{6+8}{7}$$

$$9 + 2(3) + \frac{14}{7}$$

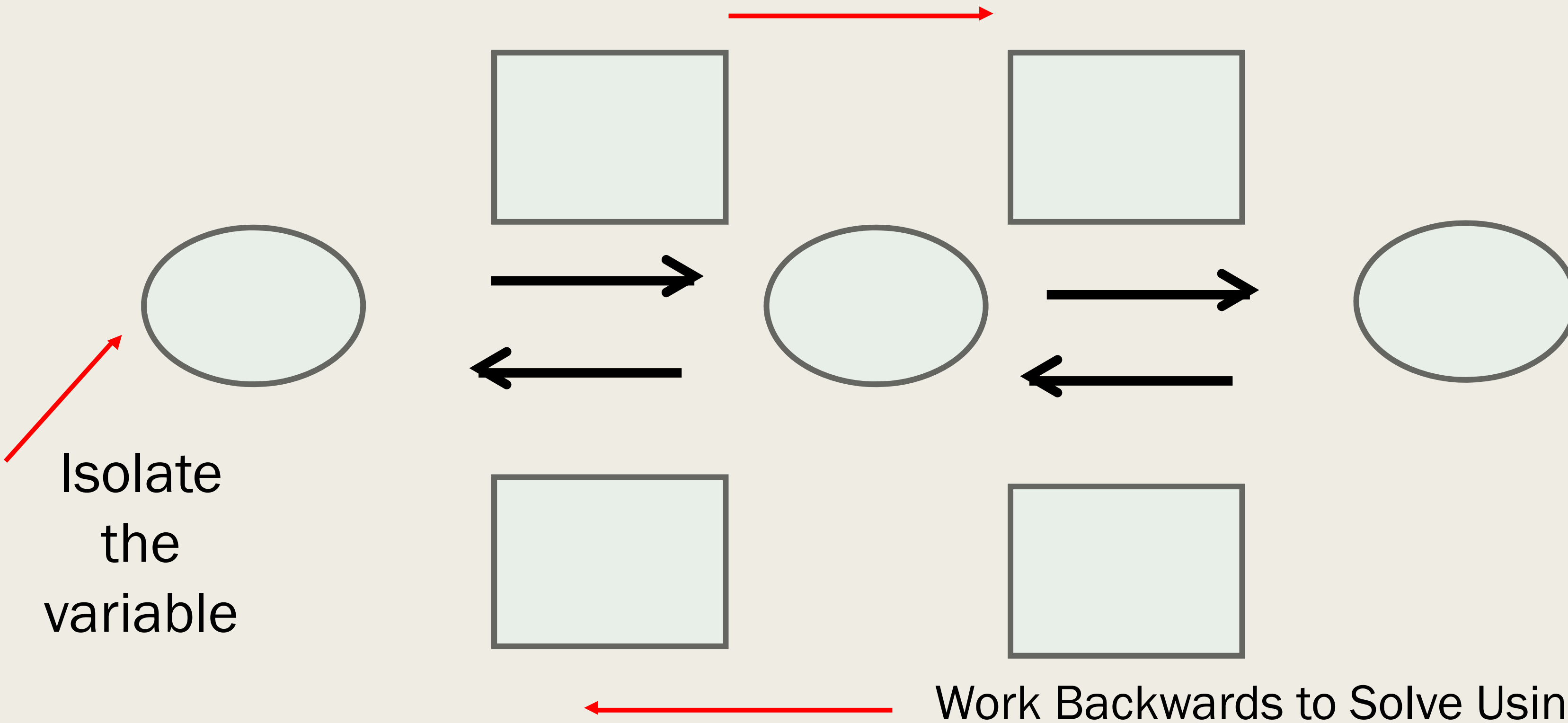
$$9 + 6 + 2$$

$$15 + 2$$

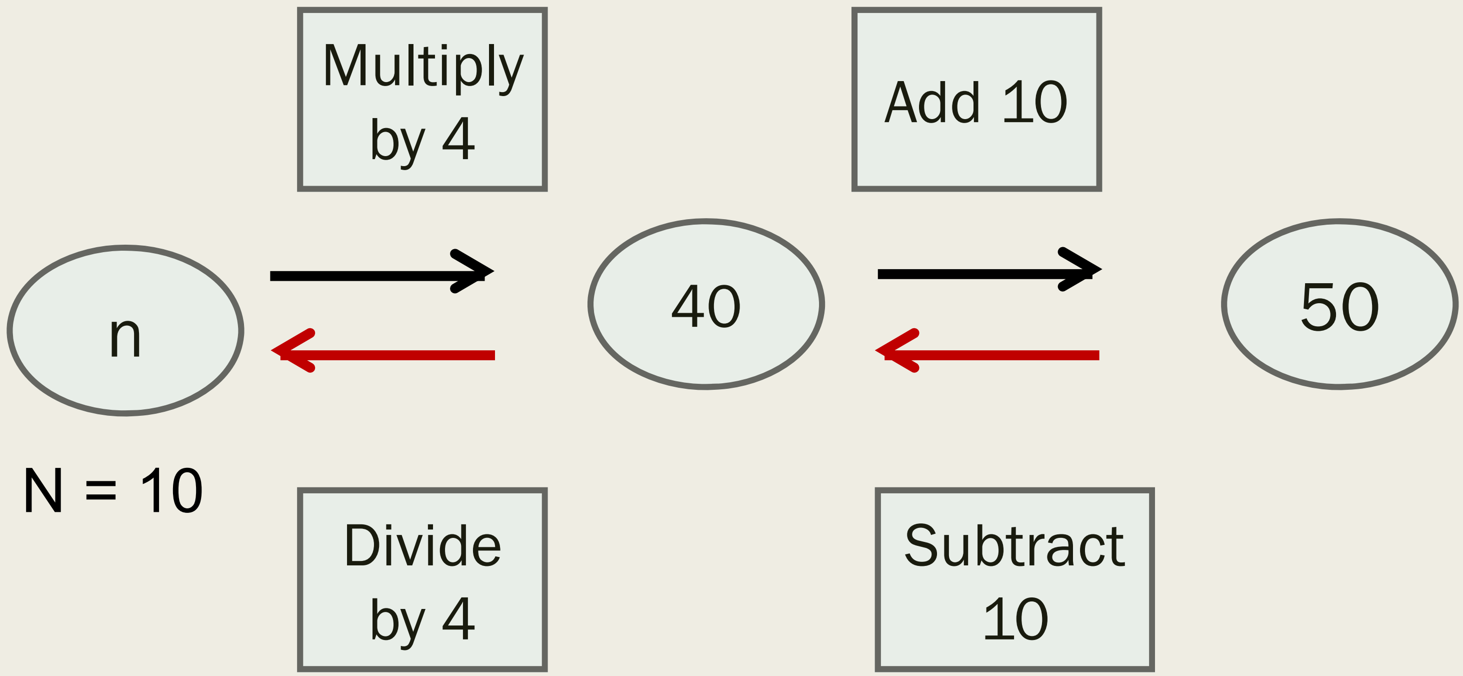
$$17$$

LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING

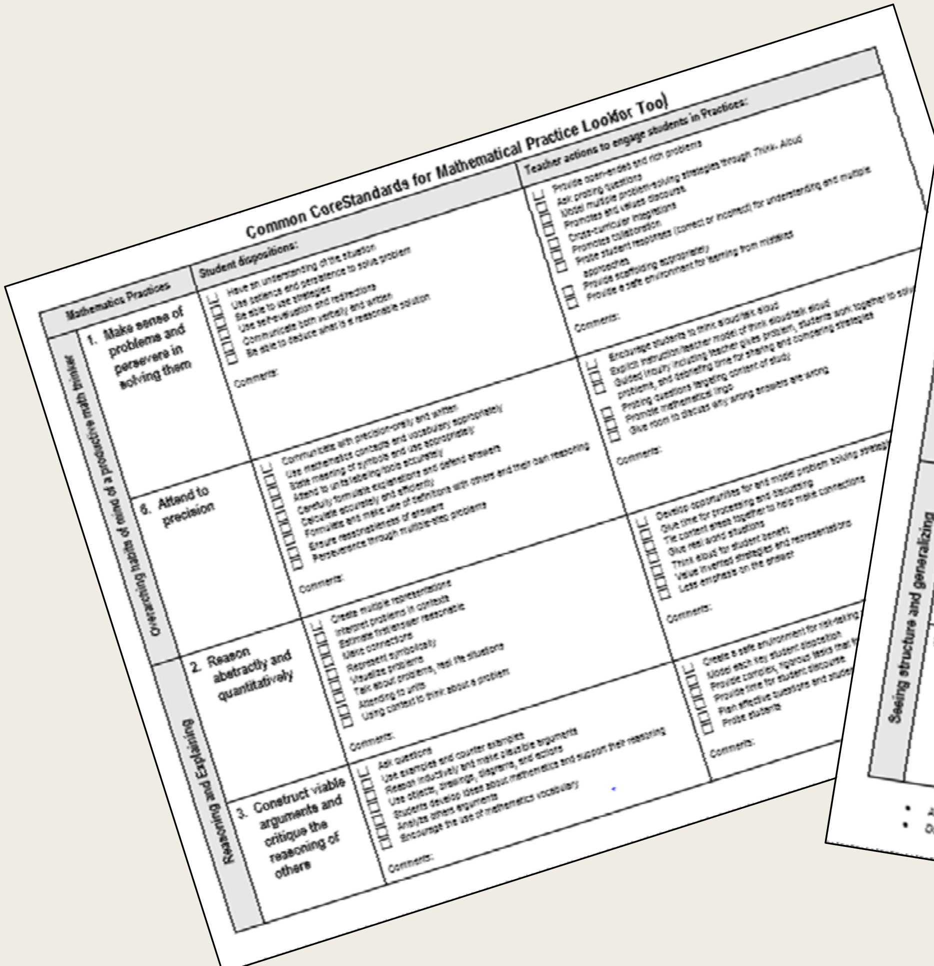
$4n + 10 = 50$

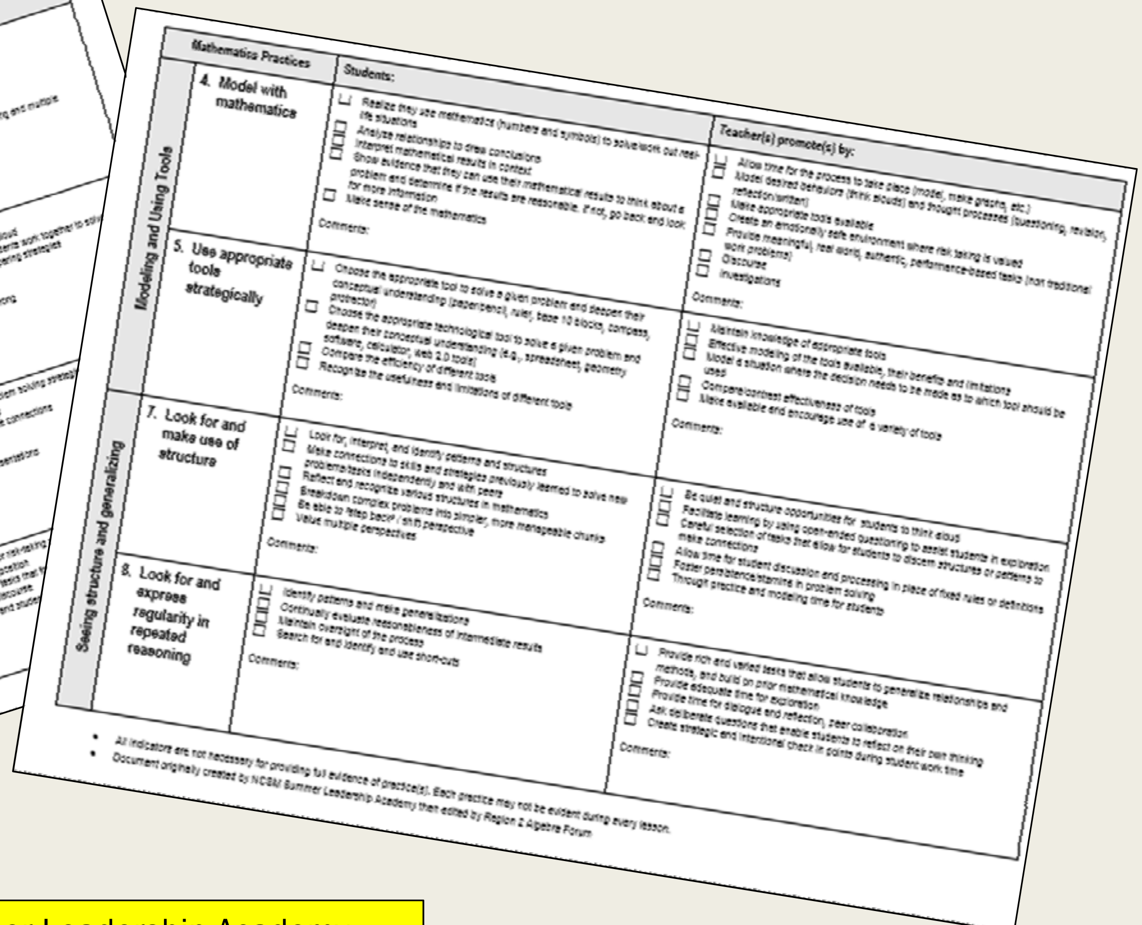


$4n + 10 = 50$



Handout – *Look-For Tool*





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